

Optimal value commitment in bilateral bargaining

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Bilateral Bargaining**

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Optimal Value Commitment in Bilateral Bargaining ¹

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Abstract

We propose a new model to study the role of commitment as a source of strategic bargaining power. Two impatient players bargain about the division of a pie under a standard bargaining protocol in discrete time with time-invariant recognition probabilities. Instantaneous utility is linear, but players discount the future by a constant factor. Before bargaining starts, a player can commit not to enter into any agreement which gives him less than some utility level. This commitment is perfectly binding initially. However, once so much time has passed that even receiving the entire pie would yield less than the committed level of utility, then the commitment becomes void. Intuitively, this simply means that no player can remain committed to something which has become impossible. We use a slight refinement of subgame-perfect equilibrium as a solution concept. If only one player can commit, then we find an immediate and efficient agreement on a division which gives the committed player (strictly) between one half and the entire pie, the exact allocation being determined uniquely by the recognition probabilities. If both players can commit sequentially before the bargaining starts, we find a unique equilibrium division with a first-mover advantage. Finally, we consider a version of the game where both players commit simultaneously before the bargaining starts. In this case, there is a range of equilibrium divisions. However, in the limit as the discount factor goes to one, no player obtains less than one third of the pie, even with arbitrarily small proposal power. Somewhat surprisingly, the equal split emerges as the only division supported by an equilibrium for any choice of the discount factor and the recognition probabilities.

KEYWORDS: Strategic Bargaining, Commitment, Subgame Perfect Equilibrium.

JEL CODES: C72, C78, D74.

1 Introduction

Two players bargain on how to divide a pie of unit size among themselves. They can only consume the pie once they have agreed on its division. Players are impatient and thus discount future consumption.

We are interested in the ability to commit as a source of bargaining power. We study this question using a notion of commitment with the following two characteristics:

First, the commitment is not expressed as a share of the pie but rather in terms of the pie's time value discounted back to the beginning of the bargaining process. The simple rationale behind this specification is that a commitment should be stated in the terms which the impatient player cares about. Such "value-committing" has been introduced to the literature earlier by Li (2007) and stands in contrast to the idea of "share-committing", which is more standard in the literature.

Second, we will assume that the commitment to a certain time value is perfectly binding as long as the pie has at least the committed value. However, as soon as so much time has elapsed that even the receipt of the entire pie would not lead to the committed value anymore, the commitment is assumed to become void. To the best of our knowledge, this notion of commitment is new to the bargaining literature.

Our assumption simply means that we do not allow a player to remain committed to something which is not feasible (anymore). This form of commitment confronts the player with the following dilemma: A high commitment becomes void soon, whereas a low commitment stays in effect for a long time.

It has long been recognized that an irrevocable (and perfectly credible) commitment would be an extremely powerful tool. In fact, if only one player can make such a commitment, the strategic situation resembles that in an ultimatum game, and the committed player captures the entire surplus – a result which seems unattractively lopsided. The literature has looked for ways to obtain more attractive or reasonable results by limiting the commitment's credibility. The standard approach which has been taken is to introduce a cost at which a commitment can be revoked. For instance, Muthoo (1992) presents a model of bargaining which generalizes the Nash (1953) demand game as well as Rubinstein's (1982) well-known alternating offer bargaining procedure. The former is seen as a polar case of irrevocable commitments, and the latter as an extreme case of revocable commitments, and the cost of revoking a commitment is used as the parameter scaling between the two. A typical result in this literature is that the player with the higher cost of revoking a commitment has an advantage, see, for instance, Muthoo (1996).

A different well-established approach is that of endogenous commitments. For instance, Fershtman and Seidmann (1993) and Li (2007) consider the possibility that rejecting a proposal commits a player not to accept any worse proposal in the future, an approach

which can be motivated by shifts in players' reference points. This approach has been extended and combined with the cost of revoking approach by Calabuig, Cunyat, and Olcina (2002). In Cunyat (2004), a player can choose the strength of his commitment before the bargaining starts.¹

In what follows, we will study the following game: One out of two players has access to the aforementioned commitment device. That player announces his commitment level. Subsequently, a potentially infinite number of bargaining rounds follows. In each such round, one of the two players is recognized as the proposer by a draw from a time-invariant probability distribution. The proposing player makes an offer and the game ends if this offer is accepted by the opponent. In case of a rejection the next round starts. However, any consumption in the next round will be discounted by a constant factor $\delta \in (0, 1)$. In line with our earlier discussion, the commitment device punishes the committed player if he accepts less than his commitment level while the pie's value is still higher than that level. But once the "*moment of truth*" where the pie's value shrinks below the commitment level has passed, no punishment is given. One interpretation is that the device punishes "*treason*" but forgives "*failure*". Agreeing to less than the commitment while the pie is still sufficiently valuable is akin to giving in to the opponent (treason, weakness), while making an agreement after the moment of truth is giving in to the facts after the failure of an excessively strong bargaining posture. One alternative commitment device would punish the player not only if he breaks his commitment before the moment of truth, but also as soon as the moment of truth is reached. After all, in the latter case it is clear that his promise cannot be fulfilled anymore. It can be shown that this seemingly stronger commitment device does not confer any bargaining power and is therefore not useful. We will see, however, that the more flexible commitment device which we propose, does confer substantial bargaining power.

Using a slight refinement of subgame-perfect Nash equilibrium, we find immediate and efficient agreement on a unique division of the pie. If the pie shrinks very rapidly, then the ability to commit is extremely valuable. The committed player can obtain almost the entire surplus even if his proposal power is close to zero. If the pie shrinks very slowly, commitment creates less bargaining power and the recognition probabilities become more important in determining the allocation of the surplus. In the limit as δ goes to one, proposal power and commitment power are "equally important" in the following sense: If one player can commit and the other player has a recognition probability close to one, then the surplus is shared almost equally.

¹In a different stream of literature, players can be of a fully rational type or of a stubborn type. Stubbornness is then a form of commitment. A typical issue within that literature is the possible incentive of a rational player to try and mimic a stubborn type. Well-known examples are Abreu and Gul (2000) and Kambe (1999). In this paper, we do not consider different types of the same player.

We present extensions of the model to games where both players can make a commitment before the bargaining starts. If they do so sequentially, then in the limit as δ goes to one, the first mover receives a share between one half and two thirds of the pie, depending on the recognition probabilities. With irrevocable and permanent commitment, one would expect the first mover to obtain the entire surplus.

We also consider the case where players make their commitments simultaneously. With irrevocable and permanent commitment, one would expect all efficient pie divisions to be supported by equilibria irrespective of the value of the discount factor. With the notion of commitment which we suggest, this is no longer true. If the discount factor is chosen sufficiently large, then we find a rather narrow range of efficient divisions which are supported by equilibria. More precisely, the share of the pie whose allocation is left unpredicted by the equilibrium concept in the limit is at most one fifth. Moreover, a player can never receive less than one third of the pie in an equilibrium with δ close to one – even with arbitrarily small proposal power. The equal split is the unique division with the property that it can be supported by an equilibrium regardless of the parameter choices for the discount factor and the recognition probabilities.

The rest of the paper is organized as follows: In the next section, we formally describe the game in which only a single player can make a commitment. In section 3, we study the bargaining stage of that game and solve for the equilibrium given the choice of commitment. In section 4, the game as a whole is solved and the optimal commitment is thus derived. In section 5, the game is extended to the case where both players can choose a commitment before the bargaining starts. Again, an analysis of the bargaining stage given the commitment levels is given. The conclusions of section 4 will be essential for this analysis. Section 6 deals with the optimal choice of commitments by both players. Section 7 concludes.

2 The game with one committed player

The player set is $N = \{1, 2\}$. The two players have a perfectly divisible pie of unit size at their disposal. They consume the pie once they have agreed on its division. Each player's instantaneous utility is equal to his consumption of pie, but future consumption is discounted by a constant and common factor $\delta \in (0, 1)$. This implies that at any time t , the players can divide among themselves a surplus of value δ^t . In the sequel, we will mean by the *surplus* the time value, discounted to time $t = 0$, of the pie to be divided.

The game G consists of a commitment stage and a bargaining stage. The game starts with the commitment stage in which a player (without loss of generality, we suppose it is player 1) chooses a level of commitment $c_1 \in [0, 1]$. The ensuing bargaining stage is set

in discrete time $t = 0, 1, \dots$. At the start of each such round t , one player is recognized as the proposer according to the probability distribution (β_1, β_2) , where $\beta_1 + \beta_2 = 1$ and $\beta_k > 0$ for both $k = 1, 2$. This player then proposes a division of the surplus, i.e. a pair $(x_1, x_2) \in \mathbb{R}_+^2$ such that $x_1 + x_2 \leq \delta^t$. If the other player rejects the proposal, round $t + 1$ starts. If the other player accepts the proposal, it is implemented and the game ends with the following payoffs for the players:

$$\begin{aligned} u_1(x_1, c_1, t) &= \begin{cases} x_1 - \lambda & \text{if } x_1 < c_1 \leq \delta^t \\ x_1 & \text{otherwise} \end{cases} \\ u_2(x_2) &= x_2 \end{aligned}$$

If players disagree forever, their payoffs are zero.

If $c_1 \leq \delta^t$, we will say that the commitment c_1 is *effective* at time t . If $c_1 > \delta^t$, we say that the commitment c_1 is *void* at time t .

The extensive form of the model admits two different interpretations. The main interpretation we use here is that a pie of size one is available but the players are impatient. Their commitments are expressed in terms of time value rather than the underlying pie itself. Another interpretation of the model is that a pie of unit size is available initially but physically shrinks by the factor δ each round, while players are indifferent to the passage of time. With this interpretation, the commitment is expressed in terms of the physical pie and expires as soon as that pie has shrunk below the commitment level.

If player 1 agrees to receive less than c_1 while the commitment is still effective, he incurs a cost λ . We are interested in commitments which are perfectly binding until they expire. Therefore, we assume that λ is large enough so that $u_1(x_1, c_1, t) < 0$ whenever $x_1 < c_1 \leq \delta^t$.² Thus, perpetual disagreement is better for player 1 than the violation of an effective commitment.

Suppose that in the game G , player 1 has chosen the commitment level c_1 at his initial decision node. Given c_1 , the game's bargaining stage will start. We refer to this bargaining stage as the *bargaining (sub-)game* $G(c_1)$, which will be analyzed in the next section. Moreover, we will denote by $G(c_1, t)$ some subgame of $G(c_1)$ which begins with the move of nature in round $t \geq 1$ of bargaining. Given a bargaining subgame $G(c_1)$ and a round t , there are many such subgames $G(c_1, t)$, all of which are, however, equivalent with regard to strategies and payoffs.³

²This is ensured for any $\lambda \geq 1$. A different appealing specification is that $\lambda(c_1) = c_1$ for any c_1 .

³For the analysis, it will be convenient that the random draw of the proposer in each round gives the game a stationary structure. Matters would be more complicated with a sequential-offers protocol such as Rubinstein (1982), although the flavor of the main (limit) results in this paper would be expected to carry over to such a setting as well.

3 Subgame perfect bargaining equilibrium

In this section, we consider the bargaining subgame $G(c_1)$ for any given commitment level c_1 , and solve for the equilibrium division of the surplus. As a solution concept, we will use a slight refinement of the well-known subgame-perfect Nash equilibrium (SPE). The purpose of the refinement is to break ties in favor of agreement. Such a behavior would follow from SPE in the entire game G , but has to be imposed exogenously when a bargaining subgame is considered in isolation. More formally, let s_i^B be the (bargaining) strategy for player $i = 1, 2$ in the game $G(c_1)$. In accordance with the usual definition of a strategy, s_i^B assigns to each history of $G(c_1)$ either a proposal to be made by player i or a decision on whether to accept or reject the opponent's current proposal. As by the standard definition, a strategy pair $(\bar{s}_i^B, \bar{s}_j^B)$ is a *subgame-perfect Nash equilibrium (SPE)* if its restriction to any subgame of $G(c_1)$ is a Nash equilibrium in that subgame. In the following definition, we introduce a refinement of this equilibrium concept.

Definition 3.1 *A strategy pair $(\bar{s}_i^B, \bar{s}_j^B)$ is a subgame-perfect bargaining equilibrium (SPBE) of $G(c_1)$ if it is an SPE and, in addition, satisfies the following conditions.*

1. *Suppose that under the profile \bar{s}^B , player $j = 1, 2$ rejects a particular proposal, say x , at some history, say h . Let \tilde{s}_j^B be the strategy which accepts the proposal x at h but agrees with strategy \bar{s}_j^B at all other histories. Then, the restriction of the profile $(\bar{s}_i^B, \bar{s}_j^B)$ to the subgame starting at h leads to a strictly greater payoff for player j in this subgame than the restriction of the profile $(\bar{s}_i^B, \tilde{s}_j^B)$.*
2. *Suppose that under the profile \bar{s}^B , there is a history h at which player $i = 1, 2$ makes a proposal, say x , which is subsequently rejected by player $j \neq i$. Suppose further that in the subgame following this rejection, the appropriate restriction of \bar{s}^B leads to a payoff of r_i for player i . Let \tilde{X} be the set of proposals which, if made by player i at history h , would subsequently be accepted by player j under the profile \bar{s}^B . Then, it holds that $r_i > \tilde{x}_i$ for all $\tilde{x} \in \tilde{X}$.*

The first condition above says that a player only rejects a proposal in SPBE when accepting the proposal would make him strictly worse off. The second condition says that a player only makes an unacceptable proposal in SPBE if this is strictly better for him than making an acceptable proposal.

In standard bargaining models without commitment, the fact that delay is costly implies that in any round there is a feasible agreement which strictly Pareto-dominates the payoff vector which would result from disagreement in that round. Under such conditions, SPE strategy profiles have the properties that agreement is reached immediately, and that a

responder accepts a proposal when indifferent between acceptance and rejection. In the model at hand, however, the commitment leads to a discontinuity in the utility function. Such a discontinuity makes it possible that in some round, some feasible agreement is strictly preferred to disagreement by one player, while the other player is indifferent, and no feasible agreement makes both players strictly better off than disagreement. In such a situation, the standard SPE concept leaves it indeterminate whether or not an agreement will be reached. With SPBE as the solution concept, agreement is ensured. It can be shown that delay on the equilibrium path is inconsistent with SPE if the entire game (including the commitment stage) is considered. In this sense, the refinement from SPE to SPBE can be seen as merely technical.

We will now formalize the idea of the “moment of truth” mentioned in the introduction. Define

$$\tau(c_1) = \begin{cases} \min\{t \in \mathbb{N} | t > \ln(c_1)/\ln(\delta)\} & \text{if } c_1 \in (0, 1] \\ 0 & \text{if } c_1 = 0 \end{cases}$$

so that in round $\tau(c_1)$ and all later rounds, the commitment c_1 is void. We will say that the commitment c_1 *expires* at time $\tau(c_1)$.

Remark 3.2 *The definition of $\tau(c_1)$ readily implies that:*

1. $\delta^{\tau(c_1)} < c_1$ for all $c_1 \in (0, 1]$;
2. $\delta c_1 \leq \delta^{\tau(c_1)}$ for all $c_1 \in [0, 1]$;
3. $\tau(c_1 \delta^t) = \tau(c_1) + t$ for all $c_1 \in (0, 1]$ and $t \in \mathbb{N}$.

These properties of $\tau(c_1)$ will be exploited repeatedly throughout the paper.

Consider a subgame $G(c_1, t)$ for any $t \geq \tau(c_1)$. Any such subgame is equivalent to a bargaining game without commitment. In fact, the only difference compared to the game in Rubinstein (1982) is that proposals are not made in an alternating fashion but that the proposer in each round is determined by a fixed recognition probability. The following lemma says that in such a subgame following the expiry of the commitment, the available surplus is divided in the proportion of the recognition probabilities.

Lemma 3.3 *In any subgame $G(c_1, \tau(c_1))$, player k 's ($k = i, j$) expected SPBE payoff is equal to $\beta_k \delta^{\tau(c_1)}$.*

Proof: Binmore (1987) proves that the SPE payoffs in such a subgame are equal to $\beta_k \delta^{\tau(c_1)}$. The corresponding SPE strategy profile conforms to our definition of an SPBE strategy profile. \square

By setting a commitment of zero, player 1 can effectively choose to play the bargaining game without commitment, recall that $\tau(0) = 0$. Lemma 3.3 implies that in an SPBE of that bargaining game, the surplus will be divided in the proportion of the recognition probabilities, as stated in the following corollary. As one would intuitively expect, the possibility to commit cannot weaken player 1's bargaining position compared to an analogous bargaining game without commitment.

Corollary 3.4 *If $c_1 = 0$, then the SPBE payoffs in $G(c_1)$ are (β_1, β_2) .*

We have earlier introduced $G(c_1, t)$ as the notation for a subgame of $G(c_1)$ which starts with the move of nature at round $t \geq 1$. Clearly, there are many such subgames, and each such subgame may in principle have more than one SPBE.

Definition 3.5 *We will say that the SPBE payoffs of $G(c_1, t)$ are essentially unique if all SPBE of all subgames of the type $G(c_1, t)$ lead to the same payoffs.*

In particular, Lemma 3.3 implies that the SPBE payoffs of $G(c_1, \tau(c_1))$ are essentially unique for any c_1 .

We now introduce the notion of a player's *aspiration*, which is crucial in the equilibrium analysis.

Definition 3.6 *Suppose that for some $t \in \mathbb{N}_0$, the SPBE payoffs of $G(c_1, t+1)$ are essentially unique and equal to (v_1^{t+1}, v_2^{t+1}) . Then, the players' aspirations at time t under the commitment c_1 are*

$$\begin{aligned}\alpha_1^t(c_1) &= \min\{x_1 \in [0, 1] | u_1(x_1, c_1, t) \geq v_1^{t+1}\} \\ \alpha_2^t(c_1) &= v_2^{t+1}(c_1)\end{aligned}$$

A player's aspiration at some point in time is that amount of surplus which the player would have to receive at that point in order to realize at least his reservation utility, that is, the expected utility from delaying the agreement to the next time period. The presence of commitment in our model creates a discontinuity in the utility function of the committed player. This discontinuity leads to a distinction between what is commonly called the reservation utility and what we have here defined as the aspiration. More formally, the above definition implies the following.

$$\alpha_1^t(c_1) = \begin{cases} v_1^{t+1}(c_1) & \text{if } t \geq \tau(c_1) \\ \max\{c_1, v_1^{t+1}(c_1)\} & \text{otherwise} \end{cases} \quad (1)$$

$$\alpha_2^t(c_1) = v_2^{t+1}(c_1) \quad (2)$$

In particular, if $\tau(c_1) \geq 1$, then the aspirations of players 1 and 2 at time $t = \tau(c_1) - 1$ are $\alpha_1^t(c_1) = c_1$ and $\alpha_2^t(c_1) = \beta_2 \delta^{\tau(c_1)}$. Intuitively, when we reason backwards from round $\tau(c_1)$ to round $\tau(c_1) - 1$, then player 1's aspiration level jumps up to c_1 due to the discontinuity which occurs in the utility function. As a consequence, the sum of the aspirations before time $\tau(c_1)$ may exceed the available surplus so that there is no scope for an agreement.

Lemma 3.7 *Suppose that for some $t \in \mathbb{N}_0$, the SPBE payoffs of $G(c_1, t+1)$ are essentially unique and equal to (v_1^{t+1}, v_2^{t+1}) . Then, the SPBE of $G(c_1, t)$ are essentially unique as well and are given by*

$$v_i^t(c_1) = \begin{cases} \alpha_i^t(c_1) + \beta_i (\delta^t - \alpha_i^t(c_1) - \alpha_j^t(c_1)) & \text{if } \alpha_i^t(c_1) + \alpha_j^t(c_1) \leq \delta^t \\ v_i^{t+1} & \text{otherwise} \end{cases} \quad (3)$$

for $i = 1, 2$ and $j \neq i$.

Proof: Suppose without loss of generality that in round t , player i is the proposer and player j is the responder. Consider the case where $\alpha_i^t + \alpha_j^t \leq \delta^t$. (In this proof, we omit the argument (c_1) from the notation.) We claim that in SPBE, agreement will be reached on the division $(\delta^t - \alpha_j^t, \alpha_j^t)$. By definition of α_j^t and the supposed essential uniqueness of SPBE of $G(c_1, t+1)$, it holds that player j weakly prefers an agreement x at t to disagreement at t if and only if $x_j \geq \alpha_j^t$. By the definition of SPBE, it follows that any proposal x will be accepted by player j at time t if and only if $x_j \geq \alpha_j^t$. In particular, the proposal $(\delta^t - \alpha_j^t, \alpha_j^t)$ is acceptable to player j . Indeed, proposing this division is weakly preferred by player i to disagreement because of the supposition that $\delta^t - \alpha_j^t \geq \alpha_i^t$. By the definition of SPBE, it follows that player i will make an acceptable proposal. Standard arguments imply that player i will not make an inefficient proposal, and not offer more than α_j^t to player j , so that the SPBE proposal is indeed exactly $(\delta^t - \alpha_j^t, \alpha_j^t)$. Taking into account that the proposer is chosen from the distribution β , we have now shown the lemma for the case where $\alpha_i^t + \alpha_j^t \leq \delta^t$. Suppose next that $\alpha_i^t + \alpha_j^t > \delta^t$. We show that no agreement is reached at t . As we have argued before, player j weakly prefers an agreement

x to disagreement if and only if $x_j \geq \alpha_j^t$. Thus, if an agreement x were reached at time t , then $x_i \leq \delta^t - \alpha_j^t < \alpha_i^t$. By definition of α_i^t , the agreement x_i gives player i a strictly lower utility than he would get if there were no agreement at t . But player i has the option to make an unacceptable proposal to player j , for instance $(\tilde{x}_i, \tilde{x}_j) = (1, 0)$. Thus, no agreement is possible in SPBE at time t , so that the reservation utilities at t are equal to the reservation utilities at $t + 1$, as desired. \square

Lemma 3.3 and Lemma 3.7 provide us with an explicit backward-induction algorithm to compute the SPBE payoffs in the game $G(c_1)$. The former lemma says that the SPBE payoffs of $G(c_1, \tau(c_1))$ are essentially unique, and also states these payoffs explicitly. But the latter lemma allows us to compute the essentially unique SPBE payoffs of $G(c_1, t)$ whenever the essentially unique SPBE payoffs of $G(c_1, t + 1)$ are known. At each step of the backward induction procedure, Equations (1)-(2) give the aspirations in round t as a function of the reservation utilities v^{t+1} . Then, Equation (3) translates the aspirations α^t into the reservation utilities v^t . Since the commitment c_1 expires at a finite time $\tau(c_1)$, we reach the payoffs v^0 after finitely many iterations and have thus computed the SPBE payoffs of the entire bargaining subgame $G(c_1)$. In this way, we arrive at Theorem 3.8, which will conclude this section. Before stating the theorem, however, we will now give some illustration of the computations carried out according to the algorithm.

Let us consider first the situation where $c_1 + \beta_2 \delta^{\tau(c_1)} > 1$. (For the purpose of this illustration, we will drop the arguments (c_1) from the notation.) Then, the backward-induction procedure begins as follows.

$$\begin{aligned} v^\tau &= (\beta_1 \delta^\tau, \beta_2 \delta^\tau) \\ \alpha^{\tau-1} &= (c_1, \beta_2 \delta^\tau) \\ v^{\tau-1} &= (\beta_1 \delta^\tau, \beta_2 \delta^\tau) = v^\tau \end{aligned}$$

The first line follows from Lemma 3.3, and the second line from Definition 3.6. Since $c_1 + \beta_2 \delta^\tau > \delta^{\tau-1}$, Lemma 3.7 implies the third line. Finally, we can observe that $v^{\tau-1} = v^\tau$.

But now, since $c_1 + \beta_2 \delta^\tau > 1 \geq \delta^t$ for any $t = 0, 1, \dots, \tau - 1$, we can iterate the argument and eventually find $v^0 = v^\tau = (\beta_1 \delta^\tau, \beta_2 \delta^\tau)$. We depict the situation on the first of the following time-lines, where the solid line segment stands for periods where agreement is possible, and the dotted line segment stands for periods where no agreement is feasible. Player 1's commitment is so high that no agreement can be reached while the commitment is effective.

Now consider the case where $c_1 > 0$ and, furthermore, $c_1 + \beta_2 \delta^{\tau(c_1)} \leq 1$. In that case, we define



$$a_1(c_1) = \max\{z \in \mathbb{N}_0 \mid \delta^z \geq c_1 + \beta_2 \delta^{\tau(c_1)}\}$$

In words, round $a_1(c_1)$ is the latest round of bargaining in which c_1 is effective and the surplus is sufficient to satisfy both players' aspirations. As in the previous case, the backward-induction algorithm is initialized by the payoffs at round τ (again, we omit the arguments (c_1)). By the same token as before, we have

$$\begin{aligned} v^\tau &= (\beta_1 \delta^\tau, \beta_2 \delta^\tau) \\ &\vdots \\ v^{a_1+1} &= (\beta_1 \delta^\tau, \beta_2 \delta^\tau) = v^\tau \end{aligned}$$

But now, by definition of a_1 , it holds that $c_1 + \beta_2 \delta^\tau \leq \delta^{a_1}$. Therefore, the second case stated in Lemma 3.7 applies, and we have

$$\begin{aligned} v_1^{a_1} &= \beta_1(\delta^{a_1} - \beta_2 \delta^\tau) + \beta_2 c_1 \\ v_2^{a_1} &= \beta_1 \beta_2 \delta^\tau + \beta_2(\delta^{a_1} - c_1) \end{aligned}$$

Since $v_1^{a_1} \geq c_1$, we now have $\alpha^{a_1-1} = v^{a_1}$. In rounds $t < a_1$, aspirations are equal to the expected payoffs in round $t+1$, and therefore they sum up to $\delta^{t+1} < \delta^t$, so that agreement is always possible. Iterating this argument yields

$$\begin{aligned} v_1^0(c_1) &= \beta_1 + \beta_2 c_1 - \beta_1 \beta_2 \delta^{\tau(c_1)} \\ v_2^0(c_1) &= \beta_2 - \beta_2 c_1 + \beta_1 \beta_2 \delta^{\tau(c_1)} \end{aligned}$$

Again, on the second time-line a solid line indicates periods in which an agreement can be reached and dotted lines indicate periods in which delay occurs because the sum of aspirations is higher than the current surplus.

The theorem below gives the SPBE payoffs in the game $G(c_1)$ as computed by the backward induction procedure in function of player 1's choice of c_1 .



Theorem 3.8 *All SPBE of the bargaining subgame $G(c_1)$ lead to a payoff for player 1 given by the following function $\pi_1(c_1)$.*

$$\pi_1(c_1) = \begin{cases} \beta_1 + \beta_2 c_1 - \beta_1 \beta_2 \delta^{\tau(c_1)} & \text{if } c_1 + \beta_2 \delta^{\tau(c_1)} \leq 1 \text{ and } c_1 > 0 \\ \beta_1 \delta^{\tau(c_1)} & \text{if } c_1 + \beta_2 \delta^{\tau(c_1)} > 1 \\ \beta_1 & \text{if } c_1 = 0 \end{cases}$$

In the first and third cases, an efficient agreement is reached immediately so that the payoff to player 2 equals $1 - \pi_1(c_1)$. Only in the second case, delay occurs, and the payoff to player 2 is equal to $\beta_2 \delta^{\tau(c_1)}$.

We emphasize that the essential uniqueness of SPBE payoffs in the bargaining subgame does not imply anything about uniqueness of SPBE strategies. For instance, in a round t such that $a_1(c_1) < t < \tau(c_1)$, no agreement is possible, since the sum of players' aspirations exceeds the available surplus. In SPBE, no agreement can therefore occur in such a round t . However, it is indeterminate which proposal is made in round t .

4 Optimal commitment

In the previous section, we have found player 1's expected SPBE payoff in the subgame $G(c_1)$ for any commitment level c_1 . Now we proceed to the analysis of the game G and ask which level of commitment player 1 will choose in equilibrium. In the game G , a pair of strategies is a *unilateral commitment equilibrium (UCE)* if player 1's choice of commitment \bar{c}_1 satisfies $\pi_1(\bar{c}_1) \geq \pi_1(c_1)$ for all $c_1 \in [0, 1]$ and the restriction of the strategy pair to the bargaining subgame $G(c_1)$ is an SPBE in that subgame.

So far, we have not used the assumption that each player has a strictly positive recognition probability. It will become essential in this section.

The content of the following two lemmas is straightforward. It is shown first that in UCE, player 1 indeed makes use of the commitment device by choosing a strictly positive c_1 . On the other hand, we show that in UCE, the commitment will be chosen sufficiently small so that an agreement can be reached before time $\tau(c_1)$.

Lemma 4.1 *In any UCE of the game G , it is true that $c_1 > 0$.*

Proof: Since $\lim_{c_1 \downarrow 0} [c_1 + \beta_2 \delta^{\tau(c_1)}] = 0$, we can find $c'_1 > 0$ sufficiently small such that $c'_1 + \beta_2 \delta^{\tau(c'_1)} \leq 1$. Then, it follows from Theorem 3.8 that $\pi_1(c') = \beta_1 + \beta_2 c'_1 - \beta_1 \beta_2 \delta^{\tau(c'_1)}$. By definition of $\tau(c_1)$, it holds that $c_1 > \delta^{\tau(c_1)}$ for any $c_1 > 0$. If $\beta_1 < 1$, this implies that $\pi_1(c'_1) > \beta_1 + \beta_2^2 \delta^{\tau(c'_1)}$, which readily implies $\pi_1(c'_1) > \beta_1$. But by Corollary 3.4, β_1 is the payoff to player 1 from the choice of $c_1 = 0$. \square

Lemma 4.2 *In any UCE of the game G , it holds that $c_1 + \beta_2 \delta^{\tau(c_1)} \leq 1$.*

Proof: Suppose not. Then, by Theorem 3.8, player 1's UCE payoff in the subgame $G(c_1)$ equals $\beta_1 \delta^{\tau(c_1)} \leq \beta_1 \delta$. Player 1 may deviate from his choice of c_1 to a commitment level of zero. In that case, a payoff of $\beta_1 > 0$ will result. Since $\beta_1 > \beta_1 \delta$, this deviation is profitable, a contradiction. \square

The two previous lemmas show that only the first case mentioned in Theorem 3.8 is relevant in a UCE of the entire game, giving rise to the following corollary.

Corollary 4.3 *In any UCE of the game G , an efficient agreement is reached immediately. The commitment level c_1 of player i satisfies $c_1 > 0$ and $c_1 + \beta_2 \delta^{\tau(c_1)} \leq 1$. The payoff to player 1 is given by*

$$\pi_1(c_1) = \beta_1 + \beta_2 c_1 - \beta_1 \beta_2 \delta^{\tau(c_1)}$$

and the payoff to player 2 equals $1 - \pi_1(c_1)$.

The expression for player 1's payoff in the above corollary can be rewritten as $\pi_1(c_1) = c_1 + \beta_1(1 - c_1 - \beta_2 \delta^{\tau(c_1)})$, and the concomitant payoff of player 2 as $\beta_2 \delta^{\tau(c_1)} + \beta_2(1 - c_1 - \beta_2 \delta^{\tau(c_1)})$. Hence, given that player 1's commitment c_1 satisfies $c_1 > 0$ and $c_1 + \beta_2 \delta^{\tau(c_1)} \leq 1$, the resulting division of the surplus can be interpreted as follows: Player 1 obtains his commitment level c_1 , player 2 his resulting aspiration $\beta_2 \delta^{\tau(c_1)}$, and the remainder is divided in the proportion of the recognition probabilities – as it would be if there were no commitment.

We now define a particular commitment level (depending on δ and β) and then show that this commitment will be chosen in UCE. Indeed, let

$$\psi_1 = \begin{cases} \delta^{\tilde{m}} & \text{if } \delta^{\tilde{m}} \geq \frac{1}{2 - \delta\beta_1} \\ 1 - \beta_2 \delta^{\tilde{m}} & \text{otherwise} \end{cases}$$

where \tilde{m} is given by

$$\tilde{m} = \min \left\{ m \in \mathbb{N}_0 \mid \delta^m \leq \frac{1}{1 + \beta_2 \delta} \right\}$$

Theorem 4.4 *In any UCE of the game G , player 1 commits to ψ_1 . Moreover, agreement is reached immediately on the division $(\varphi_1, 1 - \varphi_1)$, where $\varphi_1 = \pi_1(\psi_1)$.*

Proof:

Step 1. In UCE, agreement is reached immediately as by Corollary 4.3, under an effective commitment $c_1 > 0$ such that $c_1 + \beta_2 \delta^{\tau(c_1)} \leq 1$. Using the expression for the payoff in Corollary 4.3, the following statement is easily verified: For some $c_1 > 0$ such that $c_1 + \beta_2 \delta^{\tau(c_1)} < 1$, suppose that there exists $\varepsilon > 0$ sufficiently small so that $\tau(c_1 + \varepsilon) = \tau(c_1)$ and $c_1 + \varepsilon + \beta_2 \delta^{\tau(c_1)} \leq 1$. Then, $\pi_1(c_1 + \varepsilon) > \pi_1(c_1)$. Hence, c_1 cannot be the optimal choice of commitment. Conversely, we have shown that if c_1 is optimal, then *either* it holds that $c_1 = \delta^m$ for some $m \in \mathbb{N}_0$, *or* that $c_1 + \beta_2 \delta^{\tau(c_1)} = 1$.

Step 2. By construction of \tilde{m} , any $c_1 \geq \delta^{\tilde{m}-1}$ would violate the condition $c_1 + \beta_2 \delta^{\tau(c_1)} \leq 1$, and can thus not be optimal by Corollary 4.3. We have shown that the optimal commitment level satisfies $c_1 < \delta^{\tilde{m}-1}$.

Step 3. We show next that $c_1 \geq \delta^{\tilde{m}}$ in UCE. To see this, suppose by way of contradiction that some $c_1 < \delta^{\tilde{m}}$ is optimal. By definition of \tilde{m} , we have $c_1 + \beta_2 \delta^{\tau(c_1)} < 1$. Since the inequality is strict, the argument in Step 1 above implies that $c_1 = \delta^{m'}$ for some $m' \in \mathbb{N}_0$. But then $m' > \tilde{m}$ because $c_1 < \delta^{\tilde{m}}$. Plugging into the payoff function, we see that $\pi_1(\delta^{\tilde{m}}) > \pi_1(\delta^{m'})$. Thus, player 1 could profitably deviate from $c_1 = \delta^{m'}$ to a commitment of $\delta^{\tilde{m}}$, a contradiction. We have now established that $c_1 \in [\delta^{\tilde{m}}, \delta^{\tilde{m}-1})$ in UCE.

For clarification, we remark that $\tau(\delta^{\tilde{m}}) = \tilde{m} + 1$, whereas $\tau(c_1) = \tilde{m}$ for any $c_1 \in (\delta^{\tilde{m}}, \delta^{\tilde{m}-1})$.

Step 4. In this step, we derive a condition under which there exists some $h > 0$ so that $\delta^{\tilde{m}} + h + \beta_2 \delta^{\tilde{m}} \leq 1$ and $\pi_1(\delta^{\tilde{m}} + h) > \pi_1(\delta^{\tilde{m}})$. Rewriting the first condition, we find $h \leq 1 - (1 + \beta_2) \delta^{\tilde{m}}$. Since we are interested in $h > 0$ which satisfy the first condition, we know from Corollary 4.3 that

$\pi_1(\delta^{\tilde{m}} + h) = \beta_1 + \beta_2(\delta^{\tilde{m}} + h) - \beta_1\beta_2\delta^{\tilde{m}}$. The condition $\pi_1(\delta^{\tilde{m}} + h) > \pi_1(\delta^{\tilde{m}})$ can then be written as $(\beta_1 + \beta_2(\delta^{\tilde{m}} + h) - \beta_1\beta_2\delta^{\tilde{m}}) - (\beta_1 + \beta_2\delta^{\tilde{m}} - \beta_1\beta_2\delta^{\tilde{m}+1}) > 0$. Suitably rearranging the terms, this can be reduced to $h > \beta_1\delta^{\tilde{m}}(1 - \delta)$. We are now looking for some h such that $1 - (1 + \beta_2)\delta^{\tilde{m}} \geq h > \beta_1\delta^{\tilde{m}}(1 - \delta)$. Such h exists if and only if $\delta^{\tilde{m}} < \frac{1}{2 - \delta\beta_1}$. If $\delta^{\tilde{m}} \geq \frac{1}{2 - \delta\beta_1}$, then a commitment of $\delta^{\tilde{m}}$ is indeed optimal, as claimed in the lemma.

Step 5. Now turn to the case where $\delta^{\tilde{m}} < \frac{1}{2 - \delta\beta_1}$. In this case, we have shown in Step 4 that there does exist $h > 0$ such that $\delta^{\tilde{m}} + h + \beta_2\delta^{\tilde{m}} \leq 1$ and $\pi_1(\delta^{\tilde{m}} + h) > \pi_1(\delta^{\tilde{m}})$. Thus, $\delta^{\tilde{m}}$ is not optimal, and so it follows from the conclusion of Step 3 above that the optimal c_1 belongs to the open interval $(\delta^{\tilde{m}}, \delta^{\tilde{m}-1})$. Since this interval is open, the optimal commitment level cannot satisfy $c_1 = \delta^m$ for any $m \in \mathbb{N}_0$. But then, by the argument in Step 1, the optimal commitment level must satisfy $c_1 + \beta_2\delta^{\tau(c_1)} = 1$. Since $\tau(c_1) = \tilde{m}$ for any $c_1 \in (\delta^{\tilde{m}}, \delta^{\tilde{m}-1})$, we can conclude that the optimal commitment level is equal to $1 - \beta_2\delta^{\tilde{m}}$, as claimed in the lemma.

Step 6. We have shown that in UCE, player 1 commits to ψ_1 , as defined in the statement of the lemma. Corollary 4.3 implies immediate and efficient agreement. Moreover, the payoff to player 1 is $\pi_1(\psi_1)$, as desired. □

We will now elaborate on the most important implications of the above result and its proof, and derive a number of corollaries.

To begin with, knowing the payoff function $\pi_1(\cdot)$ and the UCE commitment level ψ_1 , we can explicitly state the UCE payoff for player 1, which we denote by φ_1 .

Corollary 4.5 *Player 1's UCE payoff φ_1 is given by*

$$\varphi_1 = \begin{cases} \beta_1 + \beta_2\delta^{\tilde{m}} - \beta_1\beta_2\delta^{\tilde{m}+1} & \text{if } \delta^{\tilde{m}} \geq \frac{1}{2 - \delta\beta_1} \\ 1 - \beta_2\delta^{\tilde{m}} & \text{otherwise} \end{cases}$$

Given that the recognition probabilities of both players and the discount factor all lie strictly between zero and one, we can see from the above expression for φ_1 that $\beta_1 < \varphi_1 < 1$. While player 1 strictly benefits from his commitment power, he never obtains the entire pie. This is intuitively clear since with the commitment device at hand here, player 2 can always choose to hold out until $\tau(c_1)$, in which case some strictly positive surplus will be left over.

Corollary 4.6 *For any configuration of the recognition probabilities and the discount factor, player 1's UCE payoff strictly exceeds β_1 , but falls short of the entire pie.*

Another implication of the above proof is that $\psi_1 > \frac{1}{2}$. To see this, suppose by way of contradiction that $\psi_1 \leq \frac{1}{2}$. Since $\frac{1}{2} + \beta_2 \delta^{\tau(\frac{1}{2})} < 1$, the arguments in Steps 1 and 3 of the proof above imply that $\psi_1 = \delta^{\tilde{m}}$. But by definition of ψ_1 , we have that $\psi_1 = \delta^{\tilde{m}}$ only if $\delta^{\tilde{m}} \geq \frac{1}{2 - \delta\beta_1}$. It follows that $\frac{1}{2} \geq \frac{1}{2 - \delta\beta_1}$, a contradiction to the assumption that δ and β_1 are strictly positive.

Corollary 4.7 *The optimal commitment level ψ_1 is strictly greater than one half, irrespective of the choices of δ and β . Moreover, since in UCE agreement is reached with the commitment effective, the UCE payoff of player 1 is greater than one half.*

Theorem 4.4 also implies that in equilibrium, player 1's commitment is chosen high enough so that disagreement in round $t = 0$ would lead to delay until the expiry of the commitment. To see this, notice that from the definition of \tilde{m} , we have $\delta^{\tilde{m}} > \frac{\delta}{1 - \beta_2 \delta}$. Since $c_1 \geq \delta^{\tilde{m}}$ by the proof of the above theorem, it holds that $c_1 + \beta_2 \delta c_1 > \delta$. Given that $\delta c_1 \leq \delta^{\tau(c_1)}$, the corollary follows.

Corollary 4.8 *In UCE, the sum of players' aspirations exceeds δ^t at any time $t = 1, \dots, \tau(\psi)$. Thus, $a_1(\psi_1) = 0$.*

Player 1 makes a commitment which is low enough to deter player 2 from holding out until time $\tau(c_1)$. But the previous corollary means that, loosely speaking, the optimal commitment is not “much” lower than a commitment which would lead to delay. More precisely, delay would result from any commitment c_1 such that $c_1 > \varphi_1$. To see this, notice that by Theorems 4.4 and 3.8 the maximum payoff for player 1 in an SPBE with immediate agreement of a game $G(c_1)$ is equal to φ_1 . This payoff is attained when $c_1 = \psi_1$. Now consider a game $G(c_1)$ for some $c_1 > \varphi_1$. Suppose that agreement is reached at $t = 0$ in this game. Since the commitment c_1 is effective at time $t = 0$, the payoff to player 1 must be at least c_1 . But since $c_1 > \varphi_1$, we have a contradiction.

Corollary 4.9 *If $c_1 > \varphi_1$, then no SPBE of the game $G(c_1)$ involves agreement at time $t = 0$.*

We have pointed out that in UCE an agreement is reached while player 1's commitment is effective, therefore we have that $\varphi_1 \geq \psi_1$. In the case where $c_1 = \delta^{\tilde{m}}$, there is some “friction” between the commitment and the resulting payoff, which arises from the fact that $\delta^{\tau(c_1)}$ changes in a “stepwise” fashion with c_1 . However, in the limit as $\delta \rightarrow 1$, these steps become ever smaller and the said friction vanishes as $c_1 - \delta^{\tau(c_1)} \rightarrow 0$. It is not surprising, then, that ψ_1 and φ_1 do converge to the same limit. More formally, the UCE commitment level ψ_1 has been defined as equal to either $\delta^{\tilde{m}}$ or $1 - \beta_2 \delta^{\tilde{m}}$. But both of these terms are arbitrarily close to $\frac{1}{1 + \beta_2}$ when δ is sufficiently close to one. Similarly, the term $\beta_1 + \beta_2 \delta^{\tilde{m}} - \beta_1 \beta_2 \delta^{\tilde{m}-1}$ used in the expression for φ_1 also converges to that same limit.

Theorem 4.10 *In the limit as $\delta \rightarrow 1$, the UCE division of the surplus converges to $(\bar{\varphi}_1, 1 - \bar{\varphi}_1) = (\frac{1}{1+\beta_2}, \frac{\beta_2}{1+\beta_2})$.*

The intuition behind the limit result is as follows. If δ is close enough to one, the surplus which remains at the moment of truth is nearly equal to the commitment. Thus, player 2 can obtain β_2 times the committed amount by delaying agreement until the moment of truth. Anticipating this, player 1 chooses the commitment just low enough to make player 2 willing to enter into an agreement immediately. Hence, the surplus is divided in the proportion $1 : \beta_2$.

Let us suppose that β_2 is very high. Given that the distribution of proposal power is very favorable to player 2, can player 1 compensate for his weakness if he is given the possibility to commit? Theorem 4.10 implies that if δ and β_2 are both close to one, player 1 can obtain about one half of the pie. Hence, if δ is large, the power of one player to commit is just sufficient to compensate for the fact that proposal power is concentrated with the other player. If δ is small, however, the ability to commit is much more powerful than that. In fact, for δ close to zero, the player who is able to commit can obtain close to the entire pie even if his proposal power is arbitrarily small. We illustrate these findings with the following numerical example.

Example 4.11 *Let $\beta_2 = 0.9$. Suppose first that the discount factor is very small, say, $\delta = 0.1$. In that case, we have that $\tilde{m} = 1$. The term $\frac{1}{2-\beta_1\delta}$ evaluates to $\frac{1}{1.99} \approx 0.5$. Since this is greater than $\delta^{\tilde{m}} = 0.1$, the optimal commitment level is given by $\psi_1 = 1 - \beta_2\delta^{\tilde{m}} = 1 - 0.1 \times 0.9 = 0.91$. Also, $\varphi_1 = 0.91$.*

Now suppose instead that $\delta = 0.9$; then $\tilde{m} = 6$. The term $\frac{1}{2-\beta_1\delta}$ evaluates to $\frac{1}{1.91} \approx 0.52$. Since this is smaller than $\delta^{\tilde{m}} = 0.9^6 \approx 0.53$, the optimal commitment level is given by $\psi_1 = \delta^{\tilde{m}} \approx 0.53$. Consequently,

$$\begin{aligned}\varphi_1 &\approx 0.1 + 0.9 \times 0.9^6 - 0.1 \times 0.9 \times 0.9^7 \\ &\approx 0.1 + 0.9 \times 0.53 - 0.09 \times 0.47 \\ &\approx 0.1 + 0.477 - 0.0423 \\ &\approx 0.5347\end{aligned}$$

This example provides a numerical illustration of the effects of large vs. small discount factors when the proposal power is prejudiced in the advantage of one player.

We see that for small δ , the implications of our notion of commitment are close to those which one would expect of an irrevocable and everlasting commitment. For large δ , however, the type of commitment which we propose leads to different results. This pattern will be observed more often in the sequel of the paper, when we deal with games in which both players have access to the commitment device.

5 Bargaining with two committed players

In this section, we will consider a bargaining (sub-)game $G(c_1, c_2)$. In this game, the two players bargain according to the protocol specified earlier for the game $G(c_1)$. That is, in each round a proposer is determined by the probability distribution β . However, in the game $G(c_1, c_2)$, both players $k = 1, 2$ are committed and thus have the following utility functions.

$$u_k(x_k, c_k, t) = \begin{cases} x_k - \lambda < 0 & \text{if } x_k < c_k \leq \delta^t \\ x_k & \text{otherwise} \end{cases}$$

To prepare the analysis, a number of concepts introduced for the game $G(c_1)$ need to be extended formally to the game $G(c_1, c_2)$. To begin with, we denote by $G(c_1, c_2, t)$ a subgame of the game $G(c_1, c_2)$ which starts with the move of nature at time t . An SPBE in such a subgame is an SPE which satisfies the two conditions set forth in Definition 3.1. We say that the SPBE payoffs of $G(c_1, c_2, t)$ are *essentially unique* if all SPBE of all subgames of the type $G(c_1, c_2, t)$ lead to the same payoffs. We denote these payoffs by $v_1^t(c_1, c_2)$ and $v_2^t(c_1, c_2)$, where we will sometimes omit the arguments (c_1, c_2) if no confusion arises. The definition of aspiration is extended to the new setting as follows.

Definition 5.1 *Suppose that for some $t \in \mathbb{N}_0$, the SPBE payoffs of $G(c_1, c_2, t+1)$ are essentially unique, and the corresponding payoff pair is (v_1^{t+1}, v_2^{t+1}) . Then, player k 's aspiration at time t under the commitments (c_1, c_2) is*

$$\alpha_k^t(c_1, c_2) = \min\{x_k \in [0, 1] \mid u_k(x_k, c_k, t) \geq v_k^{t+1}\}$$

This definition implies that

$$\alpha_k^t(c_1, c_2) = \begin{cases} v_k^{t+1}(c_1, c_2) & \text{if } t \geq \tau(c_k) \\ \max\{c_k, v_k^{t+1}(c_1, c_2)\} & \text{otherwise} \end{cases} \quad (4)$$

for both players $k = 1, 2$.

The above definition and the next two lemmas provide a backward-induction algorithm to find the SPBE payoffs in the game $G(c_1, c_2)$. Intuitively, Lemma 5.2 shows how to reason backwards from one round to the previous round, while Lemma 5.3 shows how to initialize the backward induction at the time when both commitments are void.

Lemma 5.2 *Suppose that for some $t \in \mathbb{N}_0$, the SPBE payoffs of $G(c_1, c_2, t+1)$ are essentially unique and equal to (v_1^{t+1}, v_2^{t+1}) . Then, the SPBE payoffs of $G(c_1, c_2, t)$ are essentially unique as well and given by*

$$v_k^t(c_1, c_2) = \begin{cases} \alpha_k^t(c_1, c_2) + \beta_k (\delta^t - \alpha_k^t(c_1, c_2) - \alpha_l^t(c_1, c_2)) & \text{if } \alpha_k^t(c_1, c_2) + \alpha_l^t(c_1, c_2) \leq \delta^t \\ v_k^{t+1} & \text{otherwise} \end{cases} \quad (5)$$

for any player $k = 1, 2$ and $l \neq k$.

This result follows from the proof of Lemma 3.7.

Lemma 5.3 *Let $\bar{t} = \max\{\tau(c_1), \tau(c_2)\}$. In any subgame $G(c_1, c_2, \bar{t})$, player k 's ($k = i, j$) expected SPBE payoff is equal to $\beta_k \delta^{\bar{t}}$.*

This statement corresponds to Lemma 3.3, and ensures the essential uniqueness of SPBE of $G(c_1, c_2, \bar{t})$. In previous sections, the only player who could commit was referred to as player 1. This labeling has been arbitrary. Thus, we can define ψ_k as the optimal level of commitment which player $k = 1, 2$ chooses in the game in which he is the only player who can commit. In a similar way, let $\pi_k(c_k)$ be the SPBE payoff in the bargaining subgame $G(c_k)$, and define $\varphi_k = \pi_k(\psi_k)$. For the rest of the current section, we will use i to denote the player with the highest commitment level. That is, we assume without loss of generality that $c_i \geq c_j$. Let us first consider a subgame $G(c_i, c_j, \tau(c_i))$. That is, we are interested in a subgame which starts at the time when the higher of the two commitments has become void. In such a subgame, only player j remains committed. Therefore, it seems intuitive that such a subgame is equivalent to a bargaining subgame with one committed player, as we have analyzed in Section 3. This idea is stated more formally in the following lemma.

Lemma 5.4 *In any subgame $G(c_i, c_j, \tau(c_i))$, the SPBE payoffs are equal to $\delta^{\tau(c_i)} \pi_j(c_j \delta^{-\tau(c_i)})$.*

Proof: Consider the backward-induction algorithm described by Equations (4)-(5) at any round $t \geq \tau(c_i)$. At such a round t , the Equations (4)-(5) reduce to

$$\alpha_i^t = v_i^{t+1}$$

$$\alpha_j^t = \begin{cases} v_j^{t+1} & \text{if } t \geq \tau(c_j) \\ \max\{c_j, v_j^{t+1}\} & \text{otherwise} \end{cases}$$

$$v_k^t = \begin{cases} \alpha_k^t + \beta_k(\delta^t - \alpha_i^t - \alpha_j^t) & \text{if } \alpha_i^t + \alpha_j^t \leq \delta^t \\ v_k^{t+1} & \text{otherwise, } k = i, j \end{cases}$$

These equations correspond to Equations (1)-(3) which describe the backward-induction algorithm in the game with one committed player. We can view each step of this algorithm as a function $f(v^{t+1}, c_j, \delta^t)$ which determines v^t . One can easily verify from Equations (1)-(3) that this function is linearly homogenous of degree one, that is, we have $f(\kappa v^{t+1}, \kappa c_j, \kappa \delta^t) = \kappa f(v^{t+1}, c_j, \delta^t)$ for any $\kappa > 0$. Moreover, we use the fact that $\tau(c_j \delta^t) = \tau(c_j) + t$ for $t \in \mathbb{N}_0$. One implication of this fact is that the backward induction from $\tau(c_j)$ to $\tau(c_i)$ takes as many iterative steps as the backward induction from $\tau(\tilde{c}_j)$ to round zero, where $\tilde{c}_j = c_j \delta^{-\tau(c_i)}$. Since in Section 3, the size of the initial surplus was normalized to one without loss of generality, the claim follows. \square

The backward-induction algorithm described by Equations (4)-(5) finds the essentially unique SPBE payoffs in any bargaining (sub-)game $G(c_i, c_j)$. We will write these payoffs to players i and j , respectively, as $\omega_i(c_i, c_j)$ and $\omega_j(c_i, c_j)$.

Lemma 5.4 has two important implications, which we state in the following two corollaries. In Theorem 4.4 we have found the optimal payoff for the only committed player. Since a subgame $G(c_i, c_j, \tau(c_i))$ is indeed a game with one committed player, we have the following corollary.

Corollary 5.5 *In a subgame $G(c_i, c_j, \tau(c_i))$, player j 's SPBE payoff is at most $\varphi_j \delta^{\tau(c_i)}$.*

Given any commitment level c_i chosen by player i , player j may adopt a strategy which involves making the commitment $\psi_j \delta^{\tau(c_i)}$ and not entering into any agreement before time $\tau(c_i)$.

Corollary 5.6 *It holds that $\omega_j(c_i, \psi_j \delta^{\tau(c_i)}) \geq \varphi_j \delta^{\tau(c_i)}$.*

Intuitively, given any commitment level c_i of player i , player j has the option to hold out until time $\tau(c_i)$ and then play a subgame in which he is the only committed player. In such a subgame, the analysis of Section 3 is applicable, and player j 's commitment and proposal power combined will allow him to capture a share of φ_j of the remaining surplus. However, this surplus has shrunk by a factor $\delta^{\tau(c_i)}$ in the meantime. There is a trade-off between the advantageous position of the only committed player and the cost of delay incurred to acquire this position.

We will now provide some illustration of the backward induction procedure in the game at hand, in which the notion of c_i and c_j being *close to each other* plays a crucial role. Let us suppose that c_j is sufficiently small so that $c_j + \beta_i \delta^{\tau(c_j)} \leq 1$. Then, we can define

$$a_j(c_j) = \max\{z \in \mathbb{N}_0 \mid \delta^z \geq c_j + \beta_i \delta^{\tau(c_j)}\}$$

Definition 5.7 We say that the commitments c_i and c_j are close to each other if $a_j(c_j) < \tau(c_i)$.

Lemma 5.8 Suppose $c_j + \beta_i \delta^{\tau(c_j)} \leq 1$. Then, c_i and c_j are close to each other if $c_j > \varphi_j \delta^{\tau(c_i)}$.

This statement follows from Corollary 4.9.

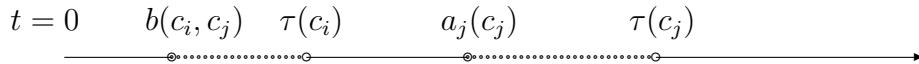
Let us suppose first that the commitments are not close to each other. We can then illustrate the situation with the following time-lines. Lemma 5.3 gives the payoffs in a subgame starting in round $\tau(c_j)$, allowing us to initialize the backward induction specified in Equations (4)-(5). Applying these equations iteratively, we find the following. Before round $\tau(c_j)$, the aspirations are $\beta_i \delta^{\tau(c_i)}$ and c_j . Thus, no agreement is possible between rounds $a_j(c_j)$ and $\tau(c_j)$, as indicated by the dotted line segment. But an agreement can be reached between time $\tau(c_i)$ and time $a_j(c_j)$. But going backwards from $\tau(c_i)$ to $\tau(c_i) - 1$, player i 's aspiration jumps to c_i while player j 's aspiration is equal to $\delta^{\tau(c_i)} \pi_j(c_j \delta^{-\tau(c_i)})$ as by Lemma 5.4. If $c_i + \delta^{\tau(c_i)} \pi_j(c_j \delta^{-\tau(c_i)}) > 1$, then no agreement can be reached before time $\tau(c_i)$, as indicated on the first time-line below.



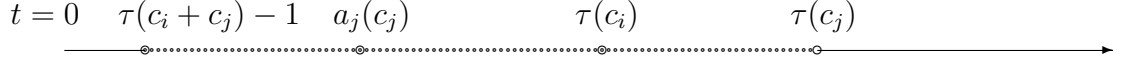
If, to the contrary, it holds that $c_i + \delta^{\tau(c_i)} \pi_j(c_j \delta^{-\tau(c_i)}) \leq 1$, then we find

$$b(c_i, c_j) := \max\{z \in \mathbb{N}_0 \mid \delta^z \geq c_i + \delta^{\tau(c_i)} \pi_j(c_j \delta^{-\tau(c_i)})\}$$

as the latest round in which agreement can be reached before $\tau(c_i)$ – a situation depicted in the next time-line.



Now consider the case where c_i and c_j are close to each other. In this case, it is not possible to reach agreement at round $a_j(c_j)$ since player i 's aspiration has jumped up to c_i already before the backward induction procedure reached round $a_j(c_j)$. Hence agreement is impossible between time $\tau(c_i + c_j) - 1$ (the latest point in time when the surplus is at least $c_i + c_j$) and time $\tau(c_j)$, as shown in the following time-line. Of course, if $c_i + c_j > 1$ and c_i and c_j are close to each other, no agreement can be reached at all until round $\tau(c_j)$.



To end the section, we provide two auxiliary results which we will use in the sequel of the paper. The first statement below says that in SPBE, the lower of the two commitments is effective at the time of the agreement unless the sum of commitments exceeds one.

Lemma 5.9 *If $c_i + c_j \leq 1$, then $\omega_j(c_i, c_j) \geq c_j$.*

Proof: Suppose not. Then, there is (c_i, c_j) such that $c_i + c_j \leq 1$ and $\omega_j(c_i, c_j) < c_j$. By definition of $\omega_j(\cdot)$, this function gives the payoff arising from some SPBE of the game $G(c_i, c_j)$. But in an SPBE, player j does not agree to less than c_j before time $\tau(c_j)$. Hence, an SPBE of $G(c_i, c_j)$ must involve delay until at least round $\tau(c_j)$. By Lemma 5.2, we have $\alpha_i^t + \alpha_j^t > \delta^t$ for all $t = 0, 1, \dots, \tau(c_j) - 1$. Notice that the assumption $c_i \geq c_j$ and the supposition $c_i + c_j \leq 1$ imply that $c_j \leq \frac{1}{2}$. But since $\delta^{\tau(c_j)} < c_j$ and $\beta_i < 1$, this in turn implies $c_j + \beta_i \delta^{\tau(c_j)} \leq 1$. Thus, $a_j(c_j)$ is well-defined and agreement can be reached in that round unless c_i and c_j are close to each other. Indeed, suppose now that c_i and c_j are close to each other. But then the aspirations in round $\tau(c_i + c_j) - 1$ are equal to the commitments c_i and c_j . Since $c_i + c_j \leq 1$, we have that $\tau(c_i + c_j) \geq 1$ and thus agreement can be reached at some round $t \in \mathbb{N}_0$, the desired contradiction. \square

In particular, if the commitments are close to each other and sum up to exactly one, then we have $\alpha_i^0 + \alpha_j^0 = 1$: The aspirations at $t = 0$ are equal to the commitments and sum up to the total surplus. By the conditions set forth in the definition of an SPBE, agreement must be reached on the division of the surplus according to the commitments.

Corollary 5.10 *If $c_i + c_j = 1$ and c_i and c_j are close to each other, then the SPBE payoffs are equal to c_i and c_j .*

6 Optimal commitment for two players

In this section, we turn to the equilibrium analysis of games where both players can commit before bargaining starts. We will consider one game in which the players make their commitments sequentially, and one in which the commitments are chosen simultaneously. Recall that the SPBE payoffs in a bargaining subgame with a given pair of commitments (c_1, c_2) are given by $\omega_1(c_1, c_2)$ and $\omega_2(c_1, c_2)$. Hence, we are effectively considering a game in which each player's action is to choose a commitment from the interval $[0, 1]$ and the payoff functions are given by $\omega_1(c_1, c_2)$ and $\omega_2(c_1, c_2)$.

In this section, we abandon the notational convention that $c_i \geq c_j$. After giving a formal definition of a (unique) best-response, we will state a number of auxiliary lemmas, which will be needed to prove the main results of this section.

Definition 6.1 *We say that the commitment \bar{c}_j is a best-response to the commitment \bar{c}_i of player $i \neq j$ if it holds that $\omega_j(\bar{c}_i, \bar{c}_j) \geq \omega_j(\bar{c}_i, c_j)$ for all $c_j \in [0, 1]$. If the inequality holds strictly for all $c_j \in [0, 1]$, then we say that \bar{c}_j is the unique best-response to \bar{c}_i .*

The next lemma claims that there is some intermediate range of commitments for player i so that player j 's best-response is to commit to the exact complement.

Lemma 6.2 *Suppose that $c_i \leq 1 - \varphi_j \delta^{\tau(c_i)}$ and $c_i \geq \varphi_i \delta^{\tau(1-c_i)}$. Then, it is a best-response for player j to choose the commitment level $c_j = 1 - c_i$. The best-response is unique if the inequalities hold strictly.*

Proof: Indeed, suppose that $c_i \leq 1 - \varphi_j \delta^{\tau(c_i)}$ and $c_i \geq \varphi_i \delta^{\tau(1-c_i)}$. By Lemma 5.8, c_i and $1 - c_i$ are close to each other, so that $\omega_i(c_i, 1 - c_i) = c_i$ and $\omega_j(c_i, 1 - c_i) = 1 - c_i$. We want to show that no choice of commitment c_j gives player j a strictly higher payoff than $1 - c_i$. If player j chooses some c_j such that the SPBE of $G(c_i, c_j)$ involves agreement before $\tau(c_i)$, the payoff to player j is bounded above by $1 - c_i$. But if j chooses c_j such that delay occurs until round $\tau(c_i)$, then by Corollary 5.5, his payoff is bounded above by $\varphi_j \delta^{\tau(c_i)} \leq 1 - c_i$, as desired. \square

In the next lemma, we consider the case where a player has chosen a commitment which is too high to fall into the range of commitments to which Lemma 6.2 applies. In that case, we show that it is optimal for the other player to hold out and delay agreement until this high commitment has expired.

Lemma 6.3 *Suppose that $c_i > 1 - \varphi_j \delta^{\tau(c_i)}$. Then, the unique best-response for player j is to choose the commitment level $c_j = \psi_j \delta^{\tau(c_i)}$.*

Proof: Suppose indeed that $1 - c_i < \varphi_j \delta^{\tau(c_i)}$. Suppose first that player j chooses some c_j such that in the essentially unique SPBE of $G(c_i, c_j)$, agreement is reached before $\tau(c_i)$. Since the commitment c_i is effective at the time of this agreement, the payoff to player j in this SPBE is at most $1 - c_i$. But by Corollary 5.6, he can obtain $\varphi_j \delta^{\tau(c_i)} > 1 - c_i$ by committing to $\psi_j \delta^{\tau(c_i)}$. Thus, a commitment level c_j can only be a best-response for player j if in an SPBE of $G(c_i, c_j)$, no agreement is made before round $\tau(c_i)$. But from Theorem 4.4 and Lemma 5.4, it follows that $\psi_j \delta^{\tau(c_i)}$ is uniquely optimal among all commitments c_j which do lead to a delay until round $\tau(c_i)$ in an SPBE of $G(c_i, c_j)$. \square

Lemma 6.4 below says the following. If player i makes a “high” commitment in the sense of Lemma 6.3 above, and if player j responds optimally to this by delaying agreement, then a unilateral deviation to a commitment of zero would be profitable for player i .

Lemma 6.4 *If c_i is such that $1 - c_i < \varphi_j \delta^{\tau(c_i)}$, then $\omega_i(c_i, \psi_j \delta^{\tau(c_i)}) < \omega_i(0, \psi_j \delta^{\tau(c_i)})$.*

Proof: Suppose that the pair of commitments (c_i, c_j) is such that $c_j = \psi_j \delta^{\tau(c_i)}$ and $1 - c_i < \varphi_j \delta^{\tau(c_i)}$. In the SPBE of a subgame $G(c_i, c_j, \tau(c_i))$, the payoffs to players i and j , respectively, will be $(1 - \varphi_j) \delta^{\tau(c_i)}$ and $\varphi_j \delta^{\tau(c_i)}$. But since $1 < c_i + \varphi_j \delta^{\tau(c_i)}$, no agreement can be reached in an SPBE at any $t < \tau(c_i)$. Thus, these payoffs are also the payoffs in the SPBE of the entire bargaining game $G(c_i, c_j)$. It holds that $c_j = \psi_j \delta^{\tau(c_i)} \leq \psi_j \delta < \psi_j$. The first inequality follows because $c_i > 0$ and thus $\tau(c_i) \geq 1$. (Suppose to the contrary that $c_i = 0$. Then, the supposition that $1 - c_i < \varphi_j \delta^{\tau(c_i)}$ reduces to $1 < \varphi_j$, a contradiction.) The second inequality follows because $\delta < 1$. Now suppose player i deviates to the commitment $\hat{c}_i = 0$, while player j remains at his commitment level $c_j < \psi_j$. The induced bargaining game $G(0, c_j)$ is equivalent to the game with one committed player, as analyzed in Section 3. Since $c_j \leq \psi_j \delta < \psi_j$, Theorem 3.8 implies that an efficient agreement is reached immediately in SPBE of $G(0, c_j)$. But, by Corollary 5.6, it holds that the payoff to player j in that agreement is at most φ_j . Thus, the payoff to player i is at least $1 - \varphi_j$. But $1 - \varphi_j > (1 - \varphi_j) \delta^{\tau(c_i)}$, so the deviation from c_i to \hat{c}_i is profitable for player i , as desired. \square

For player $i = 1, 2$, we now define a particular commitment level η_i as follows.

$$\eta_i = \max\{c_i \mid c_i + \varphi_j \delta^{\tau(c_i)} \leq 1\}$$

Loosely speaking, η_i is the maximal commitment level which is not “high” in the sense of Lemma 6.3 above. It is important to realize that $\eta_i \geq \frac{1}{2}$. In order to see this, suppose to the contrary that $\eta_i < \frac{1}{2}$. Then, it follows that $\frac{1}{2} + \varphi_j \delta^{\tau(\frac{1}{2})} > 1$. But $\varphi_j < 1$ and $\delta^{\tau(\frac{1}{2})} < \frac{1}{2}$, a contradiction.

Remark 6.5 The fact that $\eta_i \geq \frac{1}{2}$ implies that $\varphi_i \delta^{\tau(1-\eta_i)} < \frac{1}{2} \leq \eta_i$. Thus, the commitments $(\eta_i, 1 - \eta_i)$ are close to each other.

The next lemma is the last auxiliary which we need to prove the main results of the section. The lemma says that if player i commits to less than η_i , then player j can always obtain a greater payoff than $1 - \eta_i$ by choosing an appropriate commitment.

Lemma 6.6 Suppose that $c_i < \eta_i$. Then, there exists $\varepsilon > 0$ sufficiently small so that $\omega_j(c_i, 1 - \eta_i + \varepsilon) > 1 - \eta_i$.

Proof: Suppose to the contrary that there is a pair of commitments (\bar{c}_i, \bar{c}_j) such that $\bar{c}_i < \eta_i$ and $\bar{c}_j = 1 - \eta_i + \varepsilon$, but $\omega_j(\bar{c}_i, \bar{c}_j) \leq 1 - \eta_i < \bar{c}_j$. We will derive a contradiction for sufficiently small $\varepsilon > 0$. Before time $\tau(\bar{c}_j)$, player j does not agree to less than \bar{c}_j in SPBE. Thus, in any SPBE of $G(\bar{c}_i, \bar{c}_j)$, there must be delay until at least time $\tau(\bar{c}_j)$. Let us consider first the case where $\bar{c}_i > \bar{c}_j$. In SPBE, the smaller of two commitments expires only if $\bar{c}_i + \bar{c}_j > 1$ (see Lemma 5.9). This inequality is equivalent to $\bar{c}_i + 1 - \eta_i + \varepsilon > 1$. Since $\eta_i > \bar{c}_i$, we obtain the desired contradiction for sufficiently small $\varepsilon > 0$. Now consider the case where $\bar{c}_j \geq \bar{c}_i$. Since there is delay until round $\tau(\bar{c}_j)$, and since $\delta^{\tau(c_j)} < \bar{c}_j$, the payoff to player i in a subgame $G(c_i, c_j, \tau(c_j))$ is strictly smaller than $\bar{c}_j = 1 - \eta_i + \varepsilon$. In terms of the aspirations at time $\bar{t} = \tau(c_j) - 1$, we have $\alpha_i^{\bar{t}} < \bar{c}_j = 1 - \eta_i + \varepsilon$ and $\alpha_j^{\bar{t}} = \bar{c}_j = 1 - \eta_i + \varepsilon$. Summing up the aspirations yields $\alpha_i^{\bar{t}} + \alpha_i^{\bar{t}} < 2\bar{c}_j = 2(1 - \eta_i + \varepsilon)$. Using the fact that $\eta_i \geq \frac{1}{2}$, we find $\alpha_i^{\bar{t}} + \alpha_i^{\bar{t}} < 1 + 2\varepsilon$. But if $\varepsilon > 0$ is sufficiently small, then $\alpha_i^{\bar{t}} + \alpha_i^{\bar{t}} \leq 1$, in which case agreement is reached in SPBE in round \bar{t} . Since $\bar{t} < \tau(\bar{c}_j)$, we have obtained the desired contradiction. \square

We will now turn to the proof of the main results, for which the previous lemmas will be crucial. We begin with the case in which players choose their commitments sequentially. More formally, by the *sequential commitment game*, we mean the game in which first player 1 chooses a commitment level $c_1 \in [0, 1]$, then player 2 chooses a commitment level $c_2 \in [0, 1]$, and then the bargaining game $G(c_1, c_2)$ is played. Let (\bar{s}_1, \bar{s}_2) be a strategy profile in the sequential commitment game, and suppose that under this strategy profile, the players choose commitments (\bar{c}_1, \bar{c}_2) . The profile \bar{s} is a *sequential commitment equilibrium (SQCE)* if it satisfies the following conditions.

1. The restriction of \bar{s} to the bargaining subgame $G(\bar{c}_1, \bar{c}_2)$ is an SPBE in that bargaining game.
2. It holds that $\omega_2(\bar{c}_1, \bar{c}_2) \geq \omega_2(\bar{c}_1, c_2)$ for all $c_2 \in [0, 1]$.
3. There is no $(\tilde{c}_1, \tilde{c}_2) \in [0, 1] \times [0, 1]$ such that $\omega_2(\tilde{c}_1, \tilde{c}_2) \geq \omega_2(\bar{c}_1, c_2)$ for all $c_2 \in [0, 1]$, and $\omega_1(\tilde{c}_1, \tilde{c}_2) > \omega_1(\bar{c}_1, \bar{c}_2)$.

The second condition above means that player 2 responds optimally to player 1's commitment choice in any SQCE. The third condition means that player 1 chooses his commitment optimally, anticipating the response of player 2. The next theorem claims that there is a unique SQCE division of the surplus, which is immediately agreed upon.

Theorem 6.7 *In any SQCE of the sequential commitment game, an immediate and efficient agreement is reached on the division $(\eta_1, 1 - \eta_1)$.*

Proof: Suppose first that in some SQCE player 1 chooses $c_1 > \eta_1$. By Lemma 6.3, player 2 will choose $c_2 = \psi_2 \delta^{\tau(c_1)}$ in response. The resulting payoffs for player 1 will be $(1 - \varphi_2) \delta^{\tau(c_1)}$. But if player 1 had chosen a commitment of zero, then player 2's best-response would have been to choose ψ_2 , which would give player 1 the payoff of $1 - \varphi_2 > (1 - \varphi_2) \delta^{\tau(c_1)}$. Hence, we have shown that in SQCE, player 1 chooses c_1 such that $c_1 \leq \eta_1$.

Suppose that player 1 chooses $c_1 < \eta_1$. By Lemma 6.6, player 1's payoff is then strictly less than η_1 . However, he can obtain a payoff arbitrarily close to η_1 . More precisely, if player 1 chooses $c_1 = \eta_1 - \varepsilon$ for some sufficiently small $\varepsilon > 0$, then by Lemma 6.2, player 2 will choose $c_2 = 1 - c_1$ in response and thus player 1 can realize a payoff of $c_1 = \eta_1 - \varepsilon$. Thus, a commitment c_1 so that $c_1 < \eta_1$ can never be optimal.

We have now shown that $c_1 = \eta_1$ in an SQCE. By Lemma 6.2, committing to $1 - \eta_1$ is a best-response for player 2. Since η_1 and $1 - \eta_1$ are close to each other, agreement is then reached immediately on the division $(\eta_1, 1 - \eta_1)$. Finally, notice that if $\varphi_2 \delta^{\tau(c_1)} = 1 - c_1$ but $\psi_2 < \varphi_2$, then the best-response commitment $1 - \eta_1$ is not unique. It would also be a best-response for player 2 to choose $\psi_2 \delta^{\tau(c_1)}$. However, this would lead to aspirations of c_1 and $\varphi_2 \delta^{\tau(c_1)} = 1 - c_1$ at round $t = 0$. By the definition of SPBE, agreement would also be reached immediately on $(\eta_1, 1 - \eta_1)$, as desired. \square

Intuitively, Theorem 6.7 can be understood as follows. Once player 1 has chosen c_1 , player 2 always has the option to delay agreement until time $\tau(c_1)$ and then obtain the payoff $\varphi_2 \delta^{\tau(c_1)}$. Player 1 chooses his commitment just low enough so that this option becomes unattractive for player 2. Qualitatively, the result is similar to that in the game with one committed player. In that game, the player who is not committed can hold out until the commitment expires. But after the commitment has expired, the bargaining powers are given by the recognition probabilities β . In the sequential commitment case, however, once round $\tau(c_1)$ is reached, player 2's bargaining power is given by φ_2 rather than just by his recognition probability β_2 . In sum, we obtain a unique equilibrium prediction for the division of the surplus. The player who commits first has a first-mover advantage, but this advantage is weaker than the advantage of the committed over the un-committed player in the game with a single committed player. In the limit as $\delta \rightarrow 1$, we have that

$c_i - \delta^{\tau(c_i)} \rightarrow 0$, and thus η_i converges to the limit $\bar{\eta}_i = \frac{1}{1+\bar{\varphi}_j}$. But by Theorem 4.10, $\bar{\varphi}_j = \frac{1}{1+\beta_i}$. Consequently, we have the following limit result.

Theorem 6.8 *In the limit as $\delta \rightarrow 1$, the SQCE division of the surplus converges to $(\bar{\eta}_1, 1 - \bar{\eta}_1) = (\frac{1+\beta_1}{2+\beta_1}, \frac{1}{2+\beta_1})$.*

We will now turn to the version of the model where both players choose their commitments simultaneously before bargaining starts. More formally, by the *simultaneous commitment game*, we mean the game in which first players 1 and 2 simultaneously choose commitment levels $c_1 \in [0, 1]$ and $c_2 \in [0, 1]$, and then the bargaining game $G(c_1, c_2)$ is played. Let (\bar{s}_1, \bar{s}_2) be a strategy profile in the simultaneous commitment game, and (\bar{c}_1, \bar{c}_2) the commitments chosen under that profile. The profile \bar{s} is a *simultaneous commitment equilibrium (SMCE)* if its restriction to the bargaining subgame $G(\bar{c}_1, \bar{c}_2)$ is an SPBE, and if for $i = 1, 2$ and $j \neq i$, it holds that $\omega_i(\bar{c}_i, \bar{c}_j) \geq \omega_i(c_i, \bar{c}_j)$ for all $c_i \in [0, 1]$. That is, each player's commitment is a best-response to the other player's commitment. Theorem 6.9 below shows that there is a range of divisions of the surplus which can be supported by SMCE. The endpoints of this range are given by the divisions which occur in the sequential commitment equilibrium, when either player acts as the first mover.

Theorem 6.9 *A division $(x_1, x_2) \in \mathbb{R}_+^2$ of the surplus can be supported by an SMCE of the simultaneous commitment game if and only if $x_1 + x_2 = 1$ and $x_i \geq 1 - \eta_j$ for $i = 1, 2$ and $j \neq i$.*

Proof: *If:* Consider a pair of commitments (\bar{c}_1, \bar{c}_2) such that $\bar{c}_1 + \bar{c}_2 = 1$ and $\bar{c}_i \geq 1 - \eta_j$ for both $i = 1, 2$ and $j \neq i$. By definition of η_i , it follows that $\bar{c}_i \geq \varphi_i \delta^{\tau(\bar{c}_j)}$ for both $i = 1, 2$ and $j \neq i$. By Lemma 6.2, the commitments (\bar{c}_1, \bar{c}_2) are best-responses to each other. Since they are also close to each other, Corollary 5.10 implies that payoffs in the SPBE of $G(\bar{c}_1, \bar{c}_2)$ will be (\bar{c}_1, \bar{c}_2) .

Only If: We show first that in any SMCE, we have $c_i \leq \eta_i$ for both $i = 1, 2$. Suppose by way of contradiction that there is some SMCE in which the commitments are (\bar{c}_1, \bar{c}_2) and $\bar{c}_i > \eta_i$ for some $i = 1, 2$. Then, by definition of η_i , it holds that $\bar{c}_i + \varphi_j \delta^{\tau(\bar{c}_i)} > 1$ and by Lemma 6.3, it must hold that $\bar{c}_j = \psi_j \delta^{\tau(\bar{c}_i)}$ for $j \neq i$. But then, by Lemma 6.4, setting a commitment $\hat{c}_i = 0$ is a profitable deviation for player i , the desired contradiction.

We show next that player $i = 1, 2$ obtains at least a payoff of $1 - \eta_j$ in SMCE. Suppose by way of contradiction that there is some SMCE leading to payoffs (\bar{u}_1, \bar{u}_2) such that $\bar{u}_j < 1 - \eta_i$ for some $j = 1, 2$ and $i \neq j$. Again, let (\bar{c}_1, \bar{c}_2) be the commitment levels in the supposed SMCE. Lemma 6.6 then implies that $\bar{c}_i \geq \eta_i$. Suppose first that $\bar{c}_i = \eta_i$.

Since η_i and $1 - \eta_i$ are close to each other, player j could obtain the payoff of $1 - \eta_i$ by committing to it. Thus, we must have that $\bar{c}_i > \eta_i$. But we have shown before that this is not consistent with SMCE, a contradiction.

Finally, we have to show that any SMCE is efficient. Suppose by way of contradiction that there is some SMCE leading to payoffs (\bar{u}_1, \bar{u}_2) such that $\bar{u}_i \geq 1 - \eta_j$ for both $i = 1, 2$ and $j \neq i$ but $\bar{u}_1 + \bar{u}_2 < 1$. Let (\bar{c}_1, \bar{c}_2) be the commitments in the supposed SMCE. Lemma 5.2 implies that if an agreement x is reached in round t , then $x_1 + x_2 = \delta^t$. Since $\bar{u}_1 + \bar{u}_2 < 1$, the supposed SMCE must involve disagreement at $t = 0$. Disagreement continues until time $\tau(c_k)$, where we assume $c_k \geq c_l$ without loss of generality. By Corollary 5.5, we have $\bar{u}_l \leq \varphi_l \delta^{\tau(c_k)}$. But the supposition is that $\bar{u}_l \geq 1 - \eta_k$. These two inequalities imply that $\eta_k + \varphi_l \delta^{\tau(c_k)} \geq 1$. But by the definition of η_k , we have $\eta_k + \varphi_l \delta^{\tau(\eta_k)} \leq 1$. We see that $c_k \geq \eta_k$. But we have already shown that $c_k \leq \eta_k$ in an SMCE; thus we can conclude that $c_k = \eta_k$ and, moreover, $c_k + \varphi_l \delta^{\tau(c_k)} = 1$. By Lemma 6.2, player l has two potential best-responses, namely $c'_l = 1 - c_k$ and $c''_l = \psi_l \delta^{\tau(c_k)}$. In the former case, immediate agreement is reached since $c_k = \eta_k$ and $1 - c_k$ are close to each other, a contradiction. Consider the latter case. At round $t = 0$, player 2's aspiration will be $\varphi_l \delta^{\tau(c_k)}$. But since $\eta_k + \varphi_l \delta^{\tau(\eta_k)} \leq 1$ and $c_k = \eta_k$, the aspirations at time $t = 0$ sum up to exactly one. But any SMCE strategy profile induces an SPBE when restricted to the bargaining subgame, and in an SPBE agreement is reached when the aspirations sum to the available surplus. Therefore, we find immediate agreement, a contradiction. \square

Passing to the limit as $\delta \rightarrow 1$, we find the following equilibrium range.

Theorem 6.10 *If δ is sufficiently close to one, the surplus division $(x_1, x_2) \in \mathbb{R}_+^2$ can be supported by an SMCE of the simultaneous commitment game if and only if $x_1 + x_2 = 1$ and $x_i \geq \frac{1}{3 - \beta_i}$ for both $i = 1, 2$.*

One implication of Theorem 6.10 is that the division (β_1, β_2) need not be supported by an equilibrium if it is very lopsided towards one player. Put another way, if the distribution of proposal power is very disadvantageous for one player, then the commitment power can mitigate this disadvantage, even if both players have access to the commitment device simultaneously.

With the conventional notion of irrevocable share-commitments, one would expect the simultaneous commitment case to be a mere coordination problem in which any distribution can be supported by some equilibrium. In the model at hand, this is still nearly true if δ is close to zero. However, if δ is close to one, the range of equilibrium divisions shrinks considerably. More precisely, in the limit as $\delta \rightarrow 1$, the share of the surplus whose allocation is left unpredicted by SMCE is at most one fifth. Conversely, for large δ , SMCE

is sufficiently strong as a solution concept to determine eighty percent of the allocation. Moreover, we note that each player's share is bounded below by one third, even with arbitrarily low recognition probability.

Comparing the results for the sequential and simultaneous commitment games, two common points emerge.

First, our model yields predictions tantamount to what one would expect with irrevocable, everlasting commitments if δ is close to zero, but produces very different results if δ is close to one. The intuition is that with a small δ , the option to hold out until the opponent's commitment becomes void is very unattractive and hence commitment confers a lot of power.

Second, for large δ , the ability to make a commitment of the type which we propose ensures that a player will get at least one third of the surplus, even if his recognition probability is arbitrarily small. In a sense, the ability to commit is worth one third of the surplus to each player, while the value of proposal power lies in determining the allocation of the remaining third.

Since the range of SMCE divisions depends on the recognition probabilities and the discount factor, we ask whether there is any equilibrium division which is robust to changes in these parameters. The following theorem claims that the equal split of the surplus is consistent with SMCE irrespective of the aforementioned parameters. Moreover, the equal split is the only division with this property.

Theorem 6.11 *A surplus division $(x_1, x_2) \in \mathbb{R}_+^2$ can be supported by an SMCE of the simultaneous commitment game for all δ and for all β if and only if $(x_1, x_2) = (\frac{1}{2}, \frac{1}{2})$.*

Proof: *If:* Since $\eta_i \geq \frac{1}{2}$ for $i = 1, 2$, we also have $1 - \eta_i \leq \frac{1}{2}$. The claim follows from Theorem 6.9.

Only if: Consider a pie division in which player $k = 1, 2$ obtains a payoff of $\frac{1}{2} - \varepsilon$, where $\varepsilon > 0$. By Theorem 6.10, if δ is sufficiently large, SMCE requires $\frac{1}{2} - \varepsilon \geq \frac{1}{3 - \beta_k}$. This can be rewritten as $\beta_k \leq \frac{\frac{1}{2} - 3\varepsilon}{\frac{1}{2} - \varepsilon}$. But we have assumed that $\varepsilon > 0$, thus $\frac{\frac{1}{2} - 3\varepsilon}{\frac{1}{2} - \varepsilon} < 1$. Consequently, choosing δ sufficiently large and $\beta_k \in \left(\frac{\frac{1}{2} - 3\varepsilon}{\frac{1}{2} - \varepsilon}, 1\right)$ ensures that the pie division under consideration is not supported by any SMCE. \square

In the simultaneous commitment game, any division $(x, 1 - x)$ with $0 < x < 1$ can be supported by SMCE for some choice of β and δ , but Theorem 6.11 shows that the equal split is unique in being consistent with SMCE for *any* choice of these parameters. The equal split emerges as a robust focal point within the range of equilibria.

7 Conclusion

We have studied the division of a shrinking surplus through a bilateral bargaining procedure with commitment. We have proposed a new notion of commitment, which has two main characteristics. Commitment is stated in time value terms, and a commitment expires when it has been rendered infeasible by the passage of time and the ensuing shrinkage of the surplus. As a result of this modeling approach, there is a trade-off between the amount to which one commits and the duration of the commitment. Relatively moderate commitments stay in effect for longer than extreme commitments.

We have focussed our analysis on three games, namely, a game in which only one player can commit before bargaining starts, a game in which both players commit sequentially before bargaining starts, and a game in which the two players commit simultaneously prior to the bargaining stage. In each of these cases, we have applied slight refinements of subgame-perfect equilibrium as solution concepts. In an equilibrium, we always find immediate agreement on a division which corresponds to the commitments. In a sense, the agreement is already pre-determined in the commitment stage, while the bargaining stage plays the role of a threat which has a moderating influence on players' choice of a commitment.

In the game with one committed player, we have uniquely predicted the division of the surplus. The committed player obtains strictly between one half and the entire surplus, and always does strictly better than in a benchmark game without commitment. The equilibrium division of the surplus depends in a discontinuous and non-monotonic way on the discount factor. However, when the discount factor is very small, commitment confers the most bargaining power. Conversely, when the discount factor is very close to one, the recognition probabilities are relatively important as a source of bargaining power. More precisely, if the discount factor is close to one and the recognition probability of the committed player is close to zero, then the surplus is divided nearly equally. In this sense, proposal power and commitment power are "equally important" in the limit.

In the game where both players commit sequentially, we have also found a unique prediction for the division of the surplus. There is a first-mover advantage since the first mover obtains between one half and two thirds of the surplus, depending on the choice of the recognition probabilities.

In the game with simultaneous commitments, a range of surplus divisions is supported by equilibria. If the discount factor is sufficiently large, this range shrinks to at most one fifth of the entire range of feasible divisions. If the commitments were of unlimited duration, then we would expect the strategic situation to collapse to a pure coordination problem, where subgame-perfect equilibrium (or a technical refinement thereof) would have no predictive power with regard to the division. With our notion of commitment, the range

of equilibrium divisions is significantly smaller when we choose the discount factor large enough. Moreover, in the model at hand, the equal split emerges as a focal point within the range of equilibrium divisions. We have established that the equal split is the only division which is supported by an equilibrium regardless of the values of the discount factor and the recognition probabilities.

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References

- ABREU, D. AND F. GUL (2000) , “Bargaining and reputation,” *Econometrica*, 68, 85-117.
- BINMORE, K.G. (1987) , “Perfect Equilibria in Bargaining Models,” in K.Binmore and P.Dasgupta (Eds.), *The Economics of Bargaining* , Blackwell, Oxford.
- CALABUIG, V., A. CUNYAT, AND G. OLCINA (2002) , “Commitment and choice of partner in a negotiation with a deadline,” *Spanish Economic Review*, 4, 61-87.
- CUNYAT, A. (2004) , “The optimal degree of commitment in a negotiation with a deadline,” *Economic Theory*, 23, 455-465.
- FERSHTMAN, C. AND D. SEIDMANN (1993) , “Deadline effects and inefficient delay in bargaining with endogenous commitment,” *Journal of Economic Theory*, 60, 306-321.
- KAMBE, S. (1999) , “Bargaining with imperfect commitment,” *Games and Economic Behavior*, 28, 217-237.
- LI, D. (2007) , “Bargaining with history-dependent preferences,” *Journal of Economic Theory*, 136, 695-708.
- MUTHOO, A. (1992) , “Revocable commitment and sequential bargaining,” *The Economic Journal*, 102, 378-387.
- MUTHOO, A. (1996) , “A bargaining model based on the commitment tactic,” *Journal of Economic Theory*, 69, 134-152.
- NASH, J.F. (1953) , “The bargaining problem,” *Econometrica*, 18, 155-162.
- RUBINSTEIN, A.(1982) , “Perfect equilibrium in a bargaining model,” *Econometrica*, 50, 97-109.
- SHAKED, A. AND J. SUTTON (1984) , “Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model,” *Econometrica*, 52, 1351-1364.